

lib/main/ring.ath

```

1 # Ring theories
2
3 load "group"
4
5 module Ring {
6   define [+ * <0> U- -] := [Semigroup.+ MSG.* Identity.<0> Group.U- Group.-]
7   define right-distributive :=
8     (forall x y z . (x + y) * z = x * z + y * z)
9   define left-distributive :=
10    (forall x y z . z * (x + y) = z * x + z * y)
11   define theory :=
12     (make-theory ['Abelian-Group 'MSG]
13       [right-distributive left-distributive])
14 }
15
16 module Commutative-Ring {
17   define [+ * <0> U- -] := [Semigroup.+ MSG.* Identity.<0> Group.U- Group.-]
18   define *commutative := (forall x y . x * y = y * x)
19   define theory := (make-theory ['Ring] [*commutative])
20 }
21
22 module Ring-With-Identity {
23   define [+ * <0> U- - <1>] :=
24     [Semigroup.+ MSG.* Identity.<0> Group.U- Group.- MM.<1>]
25   define theory := (make-theory ['MM 'Ring] [])
26 }
27
28 module Commutative-Ring-With-Identity {
29   define [+ * <0> U- - <1>] :=
30     [Semigroup.+ MSG.* Identity.<0> Group.U- Group.- MM.<1>]
31   define theory :=
32     (make-theory ['Ring-With-Identity 'Commutative-Ring] [])
33 }

```